

1. (20 points)

Compute the derivative of the following functions:

(a) $F(x) = \int_0^{\sqrt{x}} (3t^2 - 4t + 1) dt.$

(b) $F(x) = \left(\int_0^x (6t^2 - 1) dt \right)^2.$

(c) $F(x) = \int_{x^2}^0 \sin(\cos t) dt.$

2. (20 points)

Find the Taylor series at 0 of the function

$$F(x) = \int_0^x e^{-t^2} dt.$$

3. (30 points)

Let f be a continuous function on $[a, b]$ and $\phi : [\alpha, \beta] \rightarrow \mathbb{R}$ be continuously differentiable such that $\phi(\alpha) = a$ and $\phi(\beta) = b$. Show that

$$\int_a^b f(x) dx = \int_\alpha^\beta f(\phi(t)) \phi'(t) dt.$$

4. (30 points)

Determine the area of the region enclosed by:

(a) $y = \sin(x)$, $y = \cos(x)$, $x = \frac{\pi}{2}$ and the y -axis.

(b) $x = -y^2 + 10$ and $x = (y - 2)^2$.