

HOMEWORK 3

1. (20 points)

Find the first four terms of the Taylor series for the following functions:

(a) $x^4 + x - 2$ centered at $a = 1$.

(b) $\frac{x}{\sqrt{1-x^2}}$ centered at 0.

(c) xe^{-x} centered at 0.

2. (30 points)

(a) Estimate the error in approximating $\cos(x)$ by $1 - \frac{x^2}{2}$ on the interval $[-\frac{1}{2}, \frac{1}{2}]$.

(b) Compute the value of $\sqrt[3]{7}$ with an error smaller than 10^{-4} .

3. (20 points)

Solve the following limits using Taylor Series:

(a) $\lim_{x \rightarrow 0} \frac{\ln(1-x^2)}{x^2}$.

(b) $\lim_{x \rightarrow 0} \frac{1 - \cos(\sin(x))}{x^2}$.

4. (30 points)

Prove if the following functions are Riemann integrables or not. If they are, give the value for the integral.

(a) $f(x) = c$, for c constant, on $[a, b]$.

(b) $f(x) = \begin{cases} \frac{1}{x} & \text{if } 0 < x \leq 1 \\ 0 & \text{if } x = 0. \end{cases}$ on $[0, 1]$.

(c) $f(x) = \begin{cases} 0 & \text{if } 0 < x \leq 1 \\ 1 & \text{if } x = 0. \end{cases}$ on $[0, 1]$.