

Combinatorics, winter 2024, homework 2

March 16, 2025

Please explicitly state the principles and tools of combinatorics you use in your solutions. If applicable, construct a bijection between sets to support your reasoning. While it is not necessary to show your work in detail, ensure that you identify any algebraic equations you are solving and double-check that your solutions are accurate.

1. How many ways are there to choose k squares on a $n \times n$ square board, such that no two chosen squares belong to the same column, or the same row.
2. Let n be a natural number. Find the value of the sum

$$\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{2k}.$$

3. Prove that for any positive integers n, k with $k \leq n$ we have

$$\binom{n}{k} = \binom{k-1}{k-1} + \binom{k}{k-1} + \cdots + \binom{n-1}{k-1}.$$

4. Find a combinatorial argument in terms of lattice paths showing that for any n, m, r we have

$$\binom{n}{m} \binom{m}{r} = \binom{n}{r} \binom{n-r}{m-r}.$$