Combinatorics, spring 2025, homework 1

March 9, 2025

Please explicitly state the principles of combinatorics you use in your solutions. If applicable, construct a bijection between sets to support your reasoning. While it is not necessary to show your work in detail, ensure that you identify any algebraic equations you are solving and double-check that your solutions are accurate.

1. How many 4-digit numbers are there such that the sum of their digits is strictly less than 10? For example, the sum of the digits of 1023 is 6, and the sum of the digits of 4005 is 9.

2. Find the number of ways to represent 7200 as a product of three natural numbers, such that none of them is divisible by 10. The representations that differ by the order of factors only are considered the same. E.g $8 \times 225 \times 4$ and $4 \times 225 \times 8$ are the same representations.

3. Out of 50 students, 25 can speak French, 18 can speak Russian, and 12 can speak Spanish. 8 can speak French and Russian, 6 can speak French and Spanish, and 4 can speak Russian and Spanish. 3 students speak all three languages. How many students speak none of these languages?

4. Consider all 3-letter words formed using the letters of the English alphabet. Two words are called similar if one can be obtained from the other by shifting each of its letters by the same number of positions in the alphabet. For example, the words acz and cfb are similar because each letter in acz is shifted by 2 positions to obtain the corresponding letter in cfb:

   - $a \to c$;
   - $c \to f$;
   - $z \to b$

   How many distinct 3-letter words are there, up to similarity (saying differently, how many equivalence classes of similar words are there)?