

1. Consider a real number a and let $A = \begin{pmatrix} a & 0 & 1 & 0 \\ 0 & -1 & 3a+3 & 0 \\ 0 & 0 & a & 0 \\ 0 & 1 & -3 & -1 \end{pmatrix}$.

- (a) **(15 p.)** Determine the Jordan form J of A .
 - (b) **(15 p.)** Find an invertible matrix P such that $A = PJP^{-1}$.
 - (c) **(10 p.)** Find J^n for all n in $\mathbb{N}$.
2. **(30 p.)** Let A be in $\mathbb{R}^{5 \times 5}$ with characteristic polynomial $\chi_A(x) = (x-1)^2x^3$. Find all the Jordan forms of A if A is similar to A^2 .
3. **(30 p.)** Let A be an invertible matrix in $\mathbb{C}^{3 \times 3}$ such that A is similar to A^{-1} and $\deg(m_A) = 2$. Find all possible Jordan forms of A .