

1. (30 p.) Describe all possible Jordan forms for a nilpotent linear operator on a finite-dimensional vector space V with $\dim V \leq 6$.
2. (35 p.) Find the nilpotent Jordan form and a corresponding Jordan basis for the linear operator f on $\mathbb{K}^8$ defined by the matrix

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

3. (35 p.) Find all nilpotent Jordan forms of $A \in \mathbb{R}^{13 \times 13}$ given that $\dim \ker A = 5$ and x^6 divides the minimal polynomial $m_A(x)$.