1. Let $A \in \mathbb{R}^{3\times 3}$ be a matrix such that $\chi_A(x) = (x^2-2)(x+1)$. Consider $B = -A^4 + 2A^2 - I$.

(a) **(10 p.)** Is B diagonalizable?

(b) **(5 p.)** What are the eigenvalues of B?

(c) **(5 p.)** What is $\det(B)$?

(d) **(5 p.)** What is $\operatorname{tr}(B)$?

2. Consider the matrix A in $\mathbb{R}^{4\times 4}$ given by

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}.$$

(a) **(12 p.)** For each integer d from 1 to 4, find d-dimensional subspaces that are invariant under the linear transformation f associated with A.

(b) **(13 p.)** Find an infinite collection of 1-dimensional f-invariant subspaces.

3. (20 p.) Let f be a linear operator on V. Prove that every subspace of V is f-invariant if and only if f is a scalar multiple of the identity.

4. Indicate whether the following statements are true or false. Justify your response.

(a) **(10 p.)** If the minimal polynomial of the matrix A is a divisor of $p(x) = (x^2-1)(x^2+2x)$, then A is diagonalizable.

(b) **(10 p.)** If $A \in \mathbb{R}^{3\times 3}$ is diagonalizable and $(A-3I)^2(A+2I) = 0$, then the characteristic polynomial is $\chi_A(x) = (x-3)^2(x+2)$.

(c) **(10 p.)** If u and v are eigenvectors of a linear operator f on a finite-dimensional vector space V, then $\operatorname{span}\{u+v\}$ is an f-invariant subspace of V.