1. Consider $A = \begin{pmatrix} 2 & -k & 0 \\ -1 & 2 & k \\ 0 & 1 & 2 \end{pmatrix}$ with $k \in \mathbb{R}$.

(a) **(15 p.)** For which values of k is A diagonalizable?

(b) **(15 p.)** Find the minimal polynomial of A for any value k in $\mathbb{R}$.

2. Define $A = \begin{pmatrix} 3 & -1 & -1 & -1 \\ 0 & 2 & 0 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$.

(a) **(15 p.)** Find the minimal polynomial of A.

(b) **(15 p.)** Write A^{-1} as a linear combination of $\{I, A, A^2, A^3\}$.

3. Indicate whether the following statements are true or false. Justify your response.

(a) **(10 p.)** A matrix A is diagonalizable if and only if its transpose A^T is diagonalizable.

(b) **(10 p.)** If 0 is an eigenvalue of AB, then 0 is not an eigenvalue of BA.

(c) **(10 p.)** If $A^3 - A^2 - 4A + 4I = 0$, then A is invertible and non-diagonalizable on $\mathbb{R}$.

(d) **(10 p.)** Let A and B be two matrices in $\mathbb{R}^{4\times 4}$ such that the minimal polynomials of A and B divide $p(x) = x^3 - 2x^2 + 4x$. Then A and B are similar and diagonalizable.