HOMEWORK 6

1) (30 pts) Give the following systems of linear equations in their matrix form and find the space of solutions by row-reducing said associated matrix.

(a) $\begin{cases} \frac{1}{3}x_1 + 2x_2 - 6x_3 &= 0 \\ -4x_1 + 5x_3 &= 0 \\ -3x_1 + 6x_2 - 13x_3 &= 0 \\ -\frac{7}{3}x_1 + 2x_2 - \frac{8}{3}x_3 &= 0 \end{cases}$

(b) $\begin{cases} x_1 - x_2 + 2x_3 &= 1 \\ 2x_1 + 2x_3 &= 1 \\ x_1 - 3x_2 + 4x_3 &= 2 \end{cases}$

2) (30 pts) Find the values of $k \in \mathbb{R}$ for which the following system

$$\begin{cases} 2kx_1 + 4x_2 + 2kx_3 &= 2 \\ kx_1 + (k+4)x_2 + 3x_3 &= -2 \\ -kx_1 - 2x_2 + x_3 &= 1 \\ (k+2)x_2 + (3k+1)x_3 &= -1 \end{cases}$$

is:

(i) inconsistent.

(ii) consistent independent.

(iii) consistent dependent.

3) (20 pts) Decide whether the following functions are linear maps or not. Justify.

(a) $T\colon \mathbb{R}^2 \to \mathbb{R}^2$, $T(x,y) = (\sin(x), y)$.

(b) $T\colon \mathrm{P}_2(\mathbb{R}) \to \mathrm{M}_2(\mathbb{R})$, $T(ax^2+bx+c) = \begin{pmatrix} a & b+c \\ b+c & a \end{pmatrix}$.

(c) $T\colon \mathrm{M}_2(\mathbb{R}) \to \mathbb{R}^3$, $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a+b, cd, 2a+b-3c+d)$.

(d) $\Phi : \mathrm{Hom}_{\mathbb{F}}(\mathbb{F}, V) \to V$, $\Phi(T) = T(1)$ where V is any $\mathbb{F}$-vector space.

4) (20 pts) Consider the vectors in $\mathbb{R}^2$ given by

$$\alpha_1 = (1,-1), \quad \alpha_2 = (2,-1), \quad \alpha_3 = (-3,2)$$
$$\beta_1 = (1,0), \quad \beta_2 = (0,1), \quad \beta_3 = (1,1).$$

Is is possible to define a linear map $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(\alpha_i) = \beta_i$ for $i = 1,2,3$? If so, describe $T(x,y)$.