HOMEWORK 5

1) (25 pts) Prove the following statement only by using dimensions: the only subspaces of $\mathbb{R}^2$ are $\mathbb{R}^2$, $\{(0,0)\}$ and $L = \{\lambda(a,b) \colon \lambda \in \mathbb{R}\}$ for $(a,b) \in \mathbb{R}^2$.

2) (20 pts) Decide whether the following statements are true or false. Prove or give a counterexample.

(a) Let V be a vector space with $\dim(V) = 42$. If V_1 and V_2 are subspaces of V with $\dim(V_1) = 33$ and $\dim(V_2) = 9$ such that $V = V_1 + V_2$, then $V = V_1 \oplus V_2$.

(b) Let V be a vector space with $\dim(V) = 42$. If V_1 and V_2 are subspaces of V with $\dim(V_1) = 33$ and $\dim(V_2) = 9$ such that $V_1 \cap V_2 \neq \{0\}$, then $V = V_1 \oplus V_2$.

(c) Let V_1 and V_2 be subspaces of $\mathbb{F}^8$ such that $\dim(V_1) = \dim(V_2) = 4$, then $\mathbb{F}^8 = V_1 \oplus V_2$.

(d) Let V_1 and V_2 be subspaces of $\mathbb{F}^8$ such that $\dim(V_1) = \dim(V_2) = 5$, then $V_1 \cap V_2 = \{0\}$.

3) (25 pts) Prove that the elementary row operation of interchanging two rows of a matrix can be accomplished by a finite sequence of elementary row operations of the other two types (that is, multiplying a row by a constant and/or adding a multiple of a row to another row).

4) (30 pts) Let $A = \begin{pmatrix} a & 1 & a^2 \\ 1 & a & 1 \\ 1 & a^2 & 2a \end{pmatrix}$.

(a) By applying elementary row operations, determine for which values of $a \in \mathbb{R}$ the matrix A is invertible.

(b) Give the inverse of A for the values of a found in (a).