

Workshop

Week 9

December 13, 2024

1. Let $x \in \mathbb{R}$ and $\alpha \in \mathbb{R}$, show that for the function $f(x) = x^\alpha := e^{\alpha \ln(x)}$, its derivative function is $f'(x) = \alpha x^{\alpha-1}$. Hint: Use Chain rule.
2. For $a > 0$, prove that the derivative of the exponential function $f(x) = a^x = e^{x \ln(a)}$ is $f'(x) = e^{x \ln(a)} \ln(a)$. Determine the derivative of its inverse using the Inverse Function Theorem.
3. Compute $f'(x)$ in the following cases:
 - (a) $f(x) = (x^3 - 2x + 1)^8$
 - (b) $f(x) = \cos(\sqrt{x^4 + 6})$
 - (c) $f(x) = \frac{\sqrt{1 + \sin(3x)}}{1 - x + x^5}$
 - (d) $f(x) = (x^2)^{x^3}$
 - (e) $f(x) = \sin^3(5x) \cos^2\left(\frac{x}{3}\right)$