Workshop

Week 7

November 29, 2024

1. Prove that if $\lim_{x\to 0} \dfrac{f(x)}{x} = l$ and $b \neq 0$, then $\lim_{x\to 0} \dfrac{f(bx)}{x} = bl$. What happens if $b = 0$?
2. Analyze $\lim_{x\to -\infty} \dfrac{a_n x^n + \cdots + a_0}{b_m x^m + \cdots + b_0}$
3. (a) Prove that $\lim_{x\to 0^+} f\left(\dfrac{1}{x}\right) = \lim_{x\to\infty} f(x)$.
   (b) Prove that $\lim_{x\to 0^-} f\left(\dfrac{1}{x}\right) = \lim_{x\to -\infty} f(x)$
   (c) Prove that $\lim_{x\to\infty} f(x) = \lim_{x\to -\infty} f(-x)$
4. Compute the following limits
   (a) $\lim_{x\to 1} \dfrac{\sin(x^2-1)}{x-1}$
   (b) $\lim_{x\to 0} \dfrac{\tan x}{x}$
   (c) $\lim_{x\to 0} \dfrac{x \sin x}{1-\cos x}$
   (d) $\lim_{x\to 0} \dfrac{\tan^2 x + 2x}{x + x^2}$
   (e) $\lim_{x\to 0} \dfrac{\sqrt{|x|}}{x}$
   (f) $\lim_{x\to\infty} \dfrac{\sqrt{|x|}}{x}$
   (g) $\lim_{x\to 0} \dfrac{\sin ax}{\sin bx}$
   (h) $\lim_{x\to\infty} x\left(\sqrt{x+2} - \sqrt{x}\right)$
5. Evaluate the points in the domain of f such that the function is discontinuous

(a) $f(x) = \begin{cases} x^2 \text{ if } x > 2 \\ x \text{ if } x \leq 2 \end{cases}$

(b) $f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} \text{ if } x \neq 2 \\ 1 \text{ if } x = 2 \end{cases}$