Workshop

Week 6

November 22, 2024

1. Use the $\epsilon - \delta$ definition of limit to prove the following limits:
   (a) $\lim_{x \to 2} 3x + 4 = 10$
   (b) $\lim_{x \to 0} x^3 = 0$
   (c) $\lim_{x \to 2} x^2 + x - 1 = 5$
2. Compute each limit or state that it does not exist.
   (a) $\lim_{x \to 2} \dfrac{|x-2|}{x-2}$
   (b) $\lim_{x \to 1} \dfrac{|x-2|}{x-2}$
   (c) $\lim_{x \to 9} \dfrac{9-x}{3-\sqrt{x}}$
   (d) $\lim_{x \to 1} \dfrac{1}{x^2-1} - \dfrac{1}{x^4-1}$
3. Find the following limits
   (a) $\lim_{x \to \infty} \dfrac{\sqrt{x^2+9}}{x+3}$
   (b) $\lim_{x \to \infty} \left(\dfrac{3x^2+2x+1}{x^2-3x+2} \right)^4$
   (c) $\lim_{x \to \infty} \dfrac{\left(\sqrt{x^2+1}+1\right)^2}{\sqrt[3]{x^6+1}}$
4. Use sequencial limit definition to prove that $\lim_{x \to 0} \dfrac{1}{x}$ does not exist.