

Workshop

Week 5

November 15, 2024

1. Show that if $a_n > 0$ and $\lim_{n \rightarrow \infty} na_n = l$ with $l \neq 0$, then the series $\sum a_n$ diverges.
2. Assume $a_n > 0$ and $\lim n^2 a_n$ exists. Show that $\sum a_n$ converges.
3. Decide if the following series converge or diverge

(a) $\sum_{n=1}^{\infty} \frac{16 + (-2)^n}{n2^n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{(n^2 + n)^q}$

(c) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2 - 1}}{\sqrt{n^5 + 1}}$

(d) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$

4. Prove that $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges conditionally.