Workshop

Week 3

November 1, 2024

1. (a) Consider the sequence $y_1 = 1$ and $y_{n+1} = 3 - y_n$, and set $y = \lim y_n$.
   Because (y_n) and (y_{n+1}) have the same limit taking the limit across the recursive equation gives $y = 3 - y$. Solving we conclude that $\lim y_n = 3/2$. What is wrong with this argument?

   (b) This time set $y_1 = 1$, and $y_{n+1} = 3 - \dfrac{1}{y_n}$. Find the limit of the sequence.

2. (a) Prove that the sequence defined by $x_1 = 3$ and $x_{n+1} = \dfrac{1}{4 - x_n}$ converges

   (b) Compute $\lim x_n$.

3. Decide whether the following propositions are true or false, providing a short justification for each conclusion.

   (a) If every proper subsequence of (x_n) converges, then (x_n) converges as well.

   (b) If (x_n) contains a divergent subsequence, then (x_n) diverges.

   (c) If (x_n) is monotone and contains a convergent subsequence then (x_n) converges.