Workshop

Week 2

October 25, 2024

1. Compute the supremum and infimum f the following sets. Justify.
   (a) $\left\{\frac{n^2+2}{n^2+1} : n \in \mathbb{N}\right\}$
   (b) $\left\{\frac{n}{3n+1} : n \in \mathbb{N}\right\}$
   (c) $\left\{\frac{1}{n} + (-1)^n : n \in \mathbb{N}\right\}$
   (d) $\{x \in \mathbb{Q} : q^3 \leq 3\}$
   (e) $\{x \in \mathbb{Q} : x \geq 0 \text{ and } x^2 \leq 10\}$
   (f) $\{x \in \mathbb{R} : x^2 + x + 1 \geq 0\}$
   (g) $\{x \in \mathbb{R} : x^2 + x + 1 < 0\}$
   (h) $\{x \in \mathbb{R} : |2x + 1| < 5\}$
2. Determine if the following sets are dense in $\mathbb{R}$.
   (a) $\{x \in \mathbb{R} : x < 100\}$
   (b) $\mathbb{R} - [0, 2]$
   (c) $\mathbb{Q} - \{0\}$
3. (a) Show that if $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.
   (b) Use (a) to prove that $\lim_{n\to\infty} \frac{(-1)^n}{n} = 0$.