Workshop

Week 3

November 8, 2024

1. Compute the following limit $\lim\limits_{n\to\infty}\left(\dfrac{2n+3}{2n}\right)^{3n+2}$

2. Let (x_n) and (y_n) be Cauchy sequences. Decide whether each of the following sequences is a Cauchy sequence, justifying each conclusion.

   (a) $z_n = |x_n - y_n|$

   (b) $z_n = (-1)^n x_n$

3. Decide if the following series converges:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$

   (b) $\displaystyle\sum_{n=5}^{\infty} \frac{2^n - 5^n}{9^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \sqrt[n]{n}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \pi^{n/2} \cos(n\pi)$

   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2+1}}$

   (h) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$

   (i) $\displaystyle\sum_{n=1}^{\infty} \frac{(1+\frac{1}{n})^n}{2^n}$