

# Workshop

Week 1

October 17, 2024

1. Prove:

(a)  $|a| = |-a|$

(b)  $\left| \frac{1}{a} \right| = \frac{1}{|a|}$ , if  $a \neq 0$

(c)  $\frac{|a|}{|b|} = \left| \frac{a}{b} \right|$ , if  $b \neq 0$

(d)  $|a + b + c| \leq |a| + |b| + |c|$

(e)  $|a - b| \leq |a| + |b|$

2. Let  $a, b \in \mathbb{R}$  and  $b \geq 0$ , then  $|a| \geq b$  if and only if  $a \leq -b$  or  $a \geq b$ .

3. Let  $A \subseteq \mathbb{R}$  finite and non-empty. What is  $\sup A$  and  $\inf A$ ?

4. Prove that if  $a \in A$  is a lower bound for  $A$ , then  $a = \inf A$ .

5. Let  $A$  be a nonempty set which is bounded below. It is defined the set  $B := \{b \in \mathbb{R} : b \text{ is a lower bound for } A\}$ . Show that  $\sup B = \inf A$ .

6. Show that for any bounded set  $A$ ,  $\inf A \leq \sup A$ .

7. Compute the supremum and infimum. Justify.

(a)  $\{x \in \mathbb{Q} : x \geq 0 \text{ and } x^2 \leq 10\}$

(b)  $\left\{ \frac{n^2 + 2}{n^2 + 1} : n \in \mathbb{N} \right\}$

(c)  $\left\{ \frac{3n}{n + 2} : n \in \mathbb{N} \right\}$