

Workshop

Week 12

January 3, 2025

1. Find the Taylor polynomial of order 3 at $a = 0$ for the function $f(x) = e^{e^x}$.
2. Let $f(x) = \sqrt{1+x}$. Using the Taylor polynomial of order 3 at $a = 0$, compute an approximate value for $\sqrt{2}$ and $\sqrt{9}$. Determine the remainder term.
3. (a) For which values of x can be $\sin(x)$ approximate by $x - \frac{x^3}{3!}$ with a remainder least than 10^{-4} ?
(b) Compute the remainder of approximating $\sin(x)$ by $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ for $|x| < 0.2$.
4. Use Taylor Theorem to compute the following numbers with the indicated accuracy
(a) e with a remainder least than 10^{-3}
(b) $\cos(0.1)$ with a remainder least than 10^{-4}
5. Determine the following limits using a suitable Taylor polynomial. Justify using the remainder

(a) $\lim_{x \rightarrow 0} \frac{\sin(x) - x}{x^3}$

(b) $\lim_{x \rightarrow 0} \frac{x - \sin(x)}{x(1 - \cos(3x))}$