

Tutorial

Week 3

November 1, 2024

1. Show that $\lim_{n \rightarrow \infty} \sqrt[n]{a} = 1, a > 0$.
2. Show that if $a_n \geq 0$ and $\lim a_n = a$ then $\lim \sqrt{a_n} = \sqrt{a}$.
3. Study the limit of the following sequences.
 - (a) $n - \sqrt{n^2 + 2n}$
 - (b) $\sqrt[n]{2 + (-1)^n}$
 - (c) $\sqrt[n]{\frac{1}{n!}}$
 - (d) $\sqrt[n]{2^n + 3^n}$
 - (e) $\frac{\sqrt[n]{1 + 2^n + 3^n + \dots + n^n}}{n}$
4. Let A be a nonempty bounded set and $s = \sup A$. Prove that there exists a sequence (a_n) such that $a_n \in A$ for all n and $(a_n) \rightarrow s$.