Homework 9

Week 10

December 21, 2024

1. Let $f : [a, b] \to [a, b]$ be differentiable and $|f(x)| \neq 1$ for all $x \in [a, b]$. Then f has a unique fixed point in $[a, b]$.

2. Find the number of (distinct) real roots of the equation $e^{2x} + \cos x + x = 0$.

3. Let $f, g : I \to \mathbb{R}$ differentiable on I, and $a \in I$. Show that if $f'(x) > g'(x)$ for all $x \in I$, and $f(a) = g(a)$ then $f(x) > g(x)$ for all $x > a$ and $f(x) < g(x)$ for all $x < a$.

4. Prove the following inequality

$$2 \sin x + \tan x > 3x, \text{ for all } x \in \left(0, \frac{\pi}{2}\right)$$

5. Determine for the following functions:

   - Domain
   - global, local maximum and minimum of f.
   - increasing and decreasing intervals of f.
   - An approximate graph of the functions.

   (a) $f(x) = x^x$

   (b) $f(x) = \sin(x) + \cos(x)$ $x \in [0, \pi]$

   (c) $g(x) = x - 2 \arctan(x)$

   (d) $f(x) = x^2 e^{-2x^2}$