Homework 8

Week 9

December 14, 2024

1. (25 points) Let $f_a(x) = \begin{cases} x^a & if\ x > 0 \\ 0 & if\ x \leq 0 \end{cases}$

   (a) For which values of a is f continuous at zero?

   (b) For which values of a is f differentiable at zero? In this case, is the derivative function continuous?

   (c) For which values of a is f twice-differentiable?

2. (25 points) Compute the derivate of the following functions:

   (a) $f(x) = (\sin x)^{\cos x}$

   (b) $f(x) = x^{(2x+1)^x}$

   (c) $f(x) = \sin(\sin(\sin(\sin \sin(x \cos(x)))))$

   (d) $f(x) = \arcsin(\arctan(\arccos(x^2 + 1)))$

3. (25 points) Decide if the following sentences are true or false. Justify properly.

   (a) If $f + g$ is differentiable in $x = a$ then f and g are differentiable in $x = a$.

   (b) If $f.g$ is differentiable in $x = a$ then f and g are differentiable in $x = a$.

   (c) If f is continuous in $x = a$, then $|f|$ is continuous in $x = a$.

   (d) There is a continuous function in $\mathbb{R}$ that is not differentiable in a infinite set of points.

4. (25 points) Suppose that f and g are n-times differentiable functions, then the product fg is also n-times differentiable and its n-th derivative is given by the formula:

$$(fg)^n = \sum_{k=0}^{n} \binom{n}{k} f^{(n-k)} g^{(k)}$$