

Homework 5

Week 5

November 16, 2024

1. (20 points) Prove that $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges conditionally. Hint: Use the sequence of partial sums..
2. Consider each of the following propositions. Provide short proofs for those that are true and counterexamples for those that are not.
 - (a) (8 points) If $\sum a_n$ converges absolutely, then $\sum a_n^2$ also converges absolutely.
 - (b) (8 points) If $\sum a_n$ converges and (b_n) converges, then $\sum a_n b_n$ converges.
 - (c) (9 points) If $\sum a_n$ converges conditionally, then $\sum n^2 a_n$ diverges.
3. Decide if the following series converge or diverge.
 - (a) (10 points) $\sum \frac{n(n+1)}{4^n}$
 - (b) (10 points) $\sum \frac{1}{n^{1+\frac{1}{n}}}$
 - (c) (10 points) $\sum \frac{n+4^n}{n+6^n}$
4. (25 points) Let $B = \{x \in \mathbb{Q} : 0 < x < 1\}$.
 - (a) What are the limit points of B ? Justify.
 - (b) Describe the inner set of B .
 - (c) What is the boundary of B ? Justify.