

# Homework 11

Week 12

January 4, 2025

1. (20 points) By considering appropriate Taylor polynomials, evaluate the limits below.

(a)  $\lim_{x \rightarrow 0} \frac{\ln(1+x^2)}{x \sin(x)}$

(b)  $\lim_{x \rightarrow 0} \frac{\sin^3(x)}{x(1 - \cos(x))}$

(c)  $\lim_{x \rightarrow \infty} \sqrt{1+x+x^2} - x$

Hint: Consider a change of variable.

2. (20 points) Suppose  $a_i, b_i$  are coefficients of Taylor polynomials at  $a$  of  $f$  and  $g$  respectively. Find the coefficients  $c_i$  of Taylor polynomials at  $a$  of the function  $h(x) = f(x)g(x)$ .
3. (20 points) Show that for all  $x > 0$ , the following inequality holds

$$1 + \frac{x}{2} - \frac{x^2}{8} < \sqrt{1+x} < 1 + \frac{x}{2}$$

4. (20 points) Compute the Taylor polynomial of order  $N$  at  $x = 0$  for the function  $f(x) = \frac{1}{1+x}$ .
5. (20 points) Using exercise 4, compute the Taylor polynomials of order  $N$  at  $x = 0$  of the following functions:

(a)  $\frac{1}{x-1}$

(b)  $\frac{1}{(x-1)^2}$

(c)  $\frac{(2x+3)}{(x+1)(x+3)}$

Hint: Write  $\frac{(2x+3)}{(x+1)(x+3)}$  as a sum of two rational expressions.