Homework 10

Week 11

December 27, 2024

1. (20 points) If $f : (0, \infty) \to (0, \infty)$ is differentiable at $a \in (0, \infty)$, then evaluate $\lim_{x \to a} \left(\frac{f(x)}{f(a)} \right)^{\frac{1}{\ln(x) - \ln(a)}}$

2. (20 points) Prove that $x - \frac{x^3}{3!} < \sin x$ for all $x > 0$.

3. (60 points) For a given function f determine:

   i) Domain
   ii) x-intercep and y-intercept
   iii) Symmetry
   iv) Asymptotes
   v) Increasing and decreasing intervals
   vi) Local maximum and minimum
   vii) Concavity intervals. Inflection points.
   viii) Sketch of the graph
   ix) Global maximum and minimum

   (a) $f(x) = \frac{x^2}{1 - x^2}$
   (b) $f(x) = x \ln(x)$
   (c) $f(x) = \arctan(3x) - \arctan(x)$