Workshop 1

Exercise 1

"If it rains, Abel takes an umbrella. Beatrice never takes an umbrella if it is not raining and always takes one when it is raining". What can we deduce from these statements in the different situations below? Carefully justify your answers by introducing 3 logical propositions P : "it's raining", Q : "Abel has an umbrella" and R : "Beatrice has an umbrella".

1. Abel walks with an umbrella.
2. Abel walks without an umbrella.
3. Beatrice walks with an umbrella.
4. Beatrice walks without an umbrella.
5. It is not raining.
6. It is raining.

Exercise 2

Determine which of the following propositions are true :

1. 136 is a multiple of 17 and 2 divides 167.
2. 136 is a multiple of 17 or 2 divides 167.
3. $\exists x \in \mathbb{R},\ (x+1=0 \ \text{ and } x+2=0)$.
4. $(\exists x \in \mathbb{R},\ x+1=0)$ and $(\exists x \in \mathbb{R},\ x+2=0)$.
5. $\exists x \in \mathbb{R}^*,\ \forall y \in \mathbb{R}^*,\ \forall z \in \mathbb{R}^*,\ z - xy = 0$.
6. $\forall y \in \mathbb{R}^*, \exists x \in \mathbb{R}^*,\ \forall z \in \mathbb{R}^*,\ z - xy = 0$.
7. $\forall y \in \mathbb{R}^*, \forall z \in \mathbb{R}^*,\ \exists x \in \mathbb{R}^*,\ z - xy = 0$.
8. $\exists a \in \mathbb{R},\ \forall \varepsilon > 0,\ |a| < \varepsilon$.
9. $\forall \varepsilon > 0,\ \exists a \in \mathbb{R},\ |a| < \varepsilon$.

Exercise 3

Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Write the negation of the following assertions :

1. $\forall x \in \mathbb{R},\ f(x) \neq 0$.
2. $\forall M > 0,\ \exists A > 0,\ \forall x \geq A,\ f(x) > M$.
3. $\forall x \in \mathbb{R},\ f(x) > 0 \implies x \leq 0$.

Exercise 4

Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Write with quantifiers the following assertions :

1. f is constant;
2. f is not constant;
3. f has a zero;
4. f is periodic.

Exercise 5

Determine for which reals x is the following assertion true :

$$\forall y \in [0,1],\ x \geq y \implies x \geq 2y.$$

(Hint : Consider the following cases $x \geq 2$, $x \in]0,2[$, $x = 0$ and $x < 0$.)

Exercise 6

A logical operator is said to be universal if it allows us to reconstruct all the other logical operators. In practice, it is sufficient to verify that we can reconstruct the three logical operators NOT, OR and AND to show that an operator is universal. Show that the following two operators are universal:

1. the operator NAND, defined by A NAND B = NOT(A AND B);
2. the operator NOR, defined by A NOR B = NOT(A OR B).