Workshop 3

Exercise 1

Decide if the following statements are true or false.

a) $\{\varnothing\} \in \{\varnothing, \{\varnothing\}\}$.

b) $\{\varnothing\} \subseteq \{\varnothing, \{\varnothing\}\}$.

c) $\{1, 2, 3\} \in \{1, 2, 3, \{4\}\}$.

d) $\{\{4\}\} \subseteq \{1, 2, 3, \{4\}\}$.

Exercise 2

Let the universe be all real numbers. Let $A = [3, 8), B = [2, 6], C = (1, 4)$, and $D = (5, \infty)$. Find

a) $A \cup C$.

b) $B \cap C$.

c) A^c.

d) $(A \cup C) - (B \cap D)$.

d) $A \cap B \cap C$.

Exercise 3

Let A, B, C and D be sets. Decide if the statement is True or False. If it is true, prove it. If it is false, give a counterexample.

a if $C \subseteq A, D \subseteq B$ and A and B are disjoint ($A \cap B = \varnothing$), then C and D are disjoint.

b If $(A - B) \cap (A - C) = \varnothing$, then $B \cap C = \varnothing$.

c $A - (B - C) = (A - B) - (A - C)$.

d $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

Exercise 4

Find $\mathcal{P}(A \times B)$ for $A = \{1, 2, \{1, 2\}\}$ and $B = \{q, \{t\}, \pi\}$.

Exercise 5

Find the union and intersection of each of the following families, i.e. find $\bigcup_{A \in \mathscr{A}} A$ and $\bigcap_{A \in \mathscr{A}} A$, for each $\mathscr{A}$.

a) $\mathscr{A} = \{\{1, 2, 3, 4, 5\}, \{2, 3, 4, 5, 6\}, \{3, 4, 5, 6, 7\}, \{4, 5, 6, 7, 8\}\}$

b) For each natural number n, let $A_n = \{5n, 5n + 1, 5n + 2, ..., 6n\}$ and let $\mathscr{A} = \{A_n : n \in \mathbb{N}\}$.

c) Let $\mathscr{A}$ be the set of all sets of integers that contain 10.

Exercise 6

Let $\mathscr{A} = \{A_\alpha : \alpha \in \Delta\}$ be a family of sets and let B be a set. Prove that

$$B \cap \bigcup_{\alpha \in \Delta} A_\alpha = \bigcup_{\alpha \in \Delta} (B \cap A_\alpha)$$

Exercise 7

Let $\mathscr{A} = \{A_\alpha : \alpha \in \Delta\}$ and let $\mathscr{B} = \{B_\beta : \beta \in \Gamma\}$ be families of sets. Prove that

$$\left(\bigcup_{\alpha\in\Delta} A_\alpha\right) \cap \left(\bigcup_{\beta\in\Gamma} B_\beta\right) = \bigcup_{\beta\in\Gamma}\bigcup_{\alpha\in\Delta}(A_\alpha \cap B_\beta).$$