

Exercise 1. (20 points) Write the power set of

$$A = \{\emptyset, 5, \{2, \{3\}\}\}$$

Decide if the following statements are true or false.

- (a) $\{\emptyset\} \subseteq \mathcal{P}(A)$.
- (b) $\{\emptyset\} \in \mathcal{P}(A)$.
- (c) $\{2, \{3\}\} \in A$

Exercise 2. (20 points)

- (a) Give an example of sets A, B, C such that $A \subseteq B$, $B \not\subseteq C$ and $A \subseteq C$.
- (b) Give an example of sets A, B, C such that $A \subseteq B$, $B \subseteq C$, and $C \subseteq A$.

Exercise 3. (20 points) Let the universe be the set of integers. Let E , D , $\mathbb{Z}^+$ and $\mathbb{Z}^-$ be the sets of all even, odd, positive and negative integers, respectively (0 is not positive nor negative). Find

- (a) $E - \mathbb{Z}^+$.
- (b) $(\mathbb{Z}^-)^c$.
- (c) $(D \cup \mathbb{Z}^+)^c$.

Exercise 4. (20 points) Decide if the following sentences are true for every sets A, B, C, D . Write a proof when a sentence is true, or give a counterexample when it is false.

- (a) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.
- (b) $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$.

Exercise 5. (20 points) Compute the following sets.

- (a) $\bigcup_{n \in \mathbb{N}} [-n, n]$.
- (b) $\bigcup_{n \in \mathbb{N}} [\frac{1}{n}, n]$.
- (c) $\bigcap_{r > 0} \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq r^2\}$.
- (d) $\bigcap_{n \in \mathbb{N}} \{n, n+1, n+2, \dots\}$