Homework 9

Exercise 1. In each case, give an example of denumerable sets A and B, neither of which is a subset of the other, such that

(a) $A \cap B$ is denumerable.

(b) $A \cap B$ is finite.

(c) $A - B$ is denumerable.

(d) $A - B$ is finite and nonempty.

Exercise 2. Let $\mathbb{Q}[x]$ be the set of polynomials $a_n X^n + \cdots + a_1 X + a_0$ with rational coefficients $a_n, \ldots, a_0$. Prove that $\mathbb{Q}[X]$ is denumerable.

Exercise 3. (a) Write a bijection between $[0, 1)$ and $(0, 1)$.

(b) Write a bijection between $\mathbb{R} \sim (\mathbb{R} - \mathbb{Z})$.

Exercise 4. (a) Prove that every infinite set X contains a denumerable subset.

(b) Prove that if B is denumerable and $A \subset B$ is a subset such that $B - A$ is infinite, then $B \sim (B - A)$.

(c) Prove that if X is uncountable and $A \subset X$ is a denumerable subset, then $X \sim (X - A)$.