

Exercise 1. (10 points) Write the converse and the contrapositive of each of the following sentences.

- (a) If 5 is prime then $\sqrt{2}$ is irrational.
- (b) The fish bite only when the moon is full.
- (c) To qualify for the Olympic team, a time of 3 minutes, 48 seconds or less is necessary.

Exercise 2. (10 points) Translate the following English sentences into symbolic sentences with quantifiers. The universe in each case is given in parentheses.

- (a) Not all precious stones are beautiful. (All stones)
- (b) Every nonzero real number is positive or negative. (Real numbers)
- (c) Every integer is greater than some other integer. (Integers)
- (d) Between any real number and any larger real number there is a rational number. (Real numbers)

Exercise 3. (20 points) Which of the following are true in the universe of real numbers?

- (a) $(\forall x)(\exists y)(x + y = 0)$
- (b) $(\exists x)(\forall y)(x \leq y)$
- (c) $(\forall y)(\exists!)(x = y^2)$

Exercise 4. (20 points) Prove that if x, y are real numbers then $x^2 - xy + y^2 \geq 0$.

Exercise 5. (20 points) Let $n \in \mathbb{Z}$. Prove that $n^2 + 4n + 7$ is odd if and only if n is even.

Exercise 6. (20 points) Decide if the following statements are true or false. If the statement is true, prove it. If the statement is false, give a counterexample.

Let a, b, c be integers:

- (a) If $a + b$ is even, then a and b have the same parity. (Definition: we say that a and b have the same parity if they are both even or both odd.)
- (b) If a divides bc , then a divides b and a divides c .
- (c) If a divides bc , then a divides b or a divides c .