Homework 5

Exercise 1. (25 points) In each of the following cases, compute the reminder of the division of x by m.

(a) $x = 22 \cdot 11 \cdot 2024$, $m = 3$.

(b) $x = 9^{9 \cdot 23}$, $m = 7$.

Exercise 2. (25 points)

(a) Describe all possible partitions of the set $A = \{1, 2, 3, 4\}$.

(b) How many different equivalence relations can be defined on A?

Exercise 3. (25 points) Define $\leq$ on $\mathbb{N}^2$ as follows:

$$(m, n) \leq (m', n') \quad \text{iff} \quad m|m' \ \wedge \ n|n'$$

(a) Prove that $\leq$ is a partial order.

(b) Prove that a non-empty subset of $\mathbb{N}^2$ has an upper bound if and only if it is a finite set (a set with a finite number of elements).

Exercise 4. (25 points) Let $\leq$ be the lexicographical order on $\mathbb{N}^2$, that is,

$$(m, n) \leq (m', n') \quad \text{iff} \quad m < m' \quad \text{or} \quad m = m' \ \wedge \ n \leq n'.$$

(a) Prove that $\leq$ is a total order.

(b) Find an element different from $(1, 1)$ without an immediate predecessor.

(c) Prove that any non-empty subset of $\mathbb{N}^2$ has a first element (that is, a minimum).