Exercise 1. (25 points) Let $R \subset A \times B$ and $S \subset B \times C$ be two relations. Prove that $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$.

Exercise 2. (25 points) Let $A = \{a, b, c, d\}$. Find a relation on A that is

(a) reflexive and symmetric but not transitive.

(b) reflexive and transitive but not symmetric.

(c) symmetric and transitive but not reflexive.

Exercise 3. (25 points) Define a relation $\sim$ on $\mathbb{Z} \times \mathbb{N}$ as follows:

$$(a, b) \sim (a', b') \quad \text{iff} \quad ab' = a'b$$

Prove that $\sim$ is an equivalence relation.

Exercise 4. (25 points) Name a positive integer and a negative integer that are

(a) congruent to $0 \pmod 7$ but not congruent to $0 \pmod 8$.

(b) congruent to $3 \pmod 7$ and congruent to $4 \pmod 8$.

(c) congruent to $0 \pmod 5$ and congruent to $2 \pmod 6$.

(d) congruent to $3 \pmod 4$ and congruent to $4 \pmod 6$.

If in some case it is not possible, explain why.