Algebra A – Learning Log 11

The study goal is to understand how culture influences students' mathematical conceptions and learning skills.

In this study, you will be asked to answer questions. Some of them will be mathematical questions, regarding the material of the course. In other questions you will be asked to reflect on the previous answers that you gave and about your learning process in the course.

Taking part in this study is completely voluntary. There are no right or wrong answers. The records of this study will be kept private, and will not affect your course grade. Answering the mathematical questions incorrectly will not affect your course grade.

If you decide not to take part or to skip some of the questions, it will not affect your course grade. If you decide to take part, you are free to withdraw at any time.

If you decide to participate you will benefit a **bonus of 2 points** added to your final grade (see details on the course website).

If you have any questions, please don't hesitate to contact me, at:
lital.shemen@gtiit.edu.cn

Thank you for your participation.

Notations:

$T : V \to V$ is linear transformation on a vector space V.
$\text{Ker}(T)$ is the kernel or the null space, defined as

$$\text{Ker}(T) = \{v \in V \; : \; T(v) = 0\}.$$

$\text{Im}(T)$ is the image, defined as

$$\text{Im}(T) = \{T(v) \in V \; : \; v \in V\}.$$

Answer the following problem:

Let V be vector space over a filed $\mathbb{F}$, with dimension $\dim(V) = 3$. $T : V \to V$ is linear transformation. There exists a vector $v \in V$ such that $T^2 v \neq 0$ and $T^3 v = 0$.

(a) Prove that $\{v, T^2 v, T^3 v\}$ is linearly independent.

(b) Prove that there exists a basis B of V such that $[T]_B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

(c) Find a basis and dimension for $\text{Ker}(T)$ and $\text{Im}(T)$.

Answer the following questions:

1. Rate the difficulty of the given problem.
   (1 - very easy, 9 - very hard)

   1   2   3   4   5   6   7   8   9

   What helped you solve the problem?

2. Suppose the problem is given in the exam, and is worth 20 points. How many points do you think you gained for your solution? Please explain.

3. Would you agree to participate in a short follow up interview by Zoom?

   Yes   No

Thank you for your participation!