HOMEWORK 6

1) (20 pts) Suppose $b, c \in \mathbb{R}$. Define $T : \mathbb{R}^3 \to \mathbb{R}^2$ by

$$T(x, y, z) = (2x - 4y + 3z + b, 6x + cxyz).$$

Show that T is linear if and only if $b = c = 0$.

2) (50 pts) Prove the following functions are linear maps. Find $\text{Null}(T)$ and $\text{Range}(T)$, give a basis for these subspaces and verify the dimension theorem.

(a) $T\colon \mathbb{R}^3 \to \mathbb{R}^3$, $T(x, y, z) = (x - y + 2z, 3x + y + 4z, 5x - y + 8z)$.

(b) $T\colon \mathrm{P}_3(\mathbb{R}) \to \mathrm{P}_4(\mathbb{R})$, $T(p(x)) = (x + 1)p(x)$.

(c) $T\colon \mathrm{M}_2(\mathbb{R}) \to \mathrm{P}_5(\mathbb{R})$, $T\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = (a-b)x^5 + (c+d)x^4 + (a+b)x^3 + (c+d)x^2 + (2b+3c)x + (7a-8b)$

3) (30 pts) In each case define, when possible, a linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ that satisfies the required conditions. When not possible explain why not.

(a) $\dim(\text{Range}(T)) = 2$ and $\dim(\text{Null}(T)) = 2$.

(b) $(1, 1, 0) \in \text{Range}(T)$ and $(0, 1, 1) \in \text{Null}(T)$.

(c) $(1, 1, 0) \in \text{Range}(T)$ and $(0, 1, 1), (1, 2, 1) \in \text{Null}(T)$.

(d) $\text{Range}(T) \subseteq \text{Null}(T)$.

(e) $\text{Null}(T) \subseteq \text{Range}(T)$.