HOMEWORK 5

1) (20 pts) Let V be a vector space with $\dim(V) = n$. Prove that:

(a) If $\{v_1, \ldots, v_m\} \subseteq V$ spans V, then $m \geq n$.

(b) If $\{v_1, \ldots, v_m\} \subseteq V$ is linearly independent, then $m \leq n$.

2) (20 pts) Prove that there exists a basis $\{p_0, p_1, p_2, p_3\}$ of $V = \{p \in \mathbb{F}[x] \colon \deg(p) \leq 3\}$ such that none of the polynomials p_0, p_1, p_2, p_3 has degree 2.

3) (20 pts) Suppose V is finite-dimensional, with $\dim(V) = n \geq 1$. Prove that there exist 1-dimensional subspaces $U_1, \ldots, U_n$ of V such that

$$V = U_1 \oplus \cdots \oplus U_n.$$

4) (40 pts) Let W_1 y W_2 be the following subspaces of $\mathbb{R}^6$:

$$W_1 = \{(x_1, x_2, x_3, x_4, x_5, x_6) \in \mathbb{R}^6 \colon x_1 + x_2 + x_3 = 0, x_4 + x_5 + x_6 = 0\},$$
$$W_2 = < (1, -1, 1, -1, 1, -1), (1, 0, 2, 1, 0, 0), (1, 0, -1, -1, 0, 1), (2, 1, 0, 0, 0, 0) > .$$

(a) Find a basis and the dimension of $W_1 \cap W_2$. Describe the space implicitly by equations.

(b) Find a basis and the dimension of $W_1 + W_2$. Describe the space implicitly by equations.

(c) Decide which of the following vectors are in $W_1 \cap W_2$ and which of them are in $W_1 + W_2$:

$$(1, 1, -2, -2, 1, 1), \quad (0, 0, 0, 1, 0, -1), \quad (1, 1, 1, 0, 0, 0), \quad (3, 0, 0, 1, 1, 3), \quad (-1, 2, 5, 6, 5, 4).$$