HOMEWORK 4

1) (20 pts) Let $A = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 2 & 1 & 1 & 1 \\ 3 & 2 & 0 & 3 \\ 1 & -1 & 1 & 2 \end{pmatrix}$. Determine for which values of a, the system $Ax = \begin{pmatrix} a \\ 1 \\ 0 \\ 1 \end{pmatrix}$ has a solution. For these values of a, find all possible solutions.

2) (20 pts) Describe explicitly all 3×3 row-reduced echelon matrices.

3) (30 pts) Consider the system of equations $Ax = 0$ where

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

is a 2×2 matrix over a field $\mathbb{F}$. Prove the following:

(a) If every entry of A is 0, then every (x_1, x_2) is a solution of $Ax = 0$.

(b) If $ad - bc \neq 0$, the system $Ax = 0$ has only the trivial solution $(x_1, x_2) = (0, 0)$.

(c) If $ad - bc = 0$ and some entry of A is not 0, then there exists a solution (x_1^0, x_2^0) of the system $Ax = 0$ such that (x_1, x_2) is a solution of $Ax = 0$ if and only if $(x_1, x_2) = (\lambda x_1^0, \lambda x_2^0)$ for some scalar $\lambda \in \mathbb{F}$.

4) (30 pts) Use elementary row operations to discover whether the following matrices are invertible or not. If so, give the inverse explicitly.

(a) $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{pmatrix}$ where the coefficients of A are in $\mathbb{Z}_5$.

(b) $B = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 6 \\ 5 & 0 & 2 \end{pmatrix}$ where the coefficients of B are in $\mathbb{Z}_7$.