HOMEWORK 3

1) (15 pts) Are the following subsets S of the vector space V subspaces?

(a) $S = \{ax^3 + bx^2 + cx + d \in V : a + b + c + d = 0\}$ where $V = \{p \in \mathbb{R}[x] : \deg(p) \leq 3\}$.

(b) $S = \{A \in \mathrm{M}_2(\mathbb{R}) : \det(A) = 0\}$ where $V = \mathrm{M}_2(\mathbb{R})$.

(c) $S = \{A \in \mathrm{M}_n(\mathbb{R}) : A^T = A\}$ where $V = \mathrm{M}_n(\mathbb{R})$.

2) (20 pts) Prove or give a counterexample: if U_1, U_2 and W are subspaces of V such that

$$V = U_1 \oplus W \quad \text{and} \quad V = U_2 \oplus W,$$

then $U_1 = U_2$.

3) (30 pts) Let S_1 and S_2 be subsets of a vector space V and denote the subspaces spanned by S_1 and S_2 as $\mathrm{Span}(S_1)$ and $\mathrm{Span}(S_2)$ respectively. Prove that $\mathrm{Span}(S_1 \cup S_2) = \mathrm{Span}(S_1) + \mathrm{Span}(S_2)$.

4) (35 pts) Homogeneous system of linear equations.

(a) Let W_A be the set of all solutions $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ of the homogeneous system of linear equations $Ax = 0$ where $A \in \mathrm{M}_3(\mathbb{R})$ is given by

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 3 & -1 & 0 \\ 2 & -4 & -5. \end{pmatrix}$$

Find a finite set of vectors in $\mathbb{R}^3$ which spans W_A.

(b) Let W_B be the set of all solutions $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ of the homogeneous system of linear equations $Bx = 0$ where $B \in \mathrm{M}_3(\mathbb{R})$ is given by

$$A = \begin{pmatrix} 1 & -3 & 1 \\ 2 & -6 & 2 \\ 3 & -9 & 3. \end{pmatrix}$$

Find a finite set of vectors in $\mathbb{R}^3$ which spans W_B.