Homework 1

1) (20 pts) Let $\mathbb{F}$ be a field. Prove the following properties:

(a) If $a.b = 0$ for $a, b \in \mathbb{F}$, then $a = 0$ or $b = 0$.

(b) The additive identity 0 is unique.

(c) If $(-a).(-b) = a.b$ for every $a, b \in \mathbb{F}$.

2) (34 pts) Define addition $\oplus$ and multiplication $\otimes$ on $\mathbb{R}$ as follows:

$$a \oplus b = a + b + 4$$
$$a \otimes b = 2ab$$

Check whether or not $\mathbb{R}$ with the addition $\oplus$ and the multiplication $\otimes$ satisfies each of the field axioms.

3) (30 pts) Let $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix}$. Find a matrix D such that $CD - AB = 0$.

4) (16 pts) Consider the fields $\mathbb{Z}_7$ and $\mathbb{Z}_{17}$.

(a) Find $3.6 + (4^{-1}) + 2.(-3) + 5$ in $\mathbb{Z}_7$.

(b) Find $3.6 + (4^{-1}) + 2.(-3) + 5$ in $\mathbb{Z}_{17}$.